5.14) Hallamos DVS de A:

$$A^T A = \begin{bmatrix} 11 & 1 \\ 1 & 11 \end{bmatrix} \qquad P(\lambda) = (11-\lambda)^2 - 1$$

<u>Autovalores</u> $\begin{cases} \lambda_1 = 12 \\ \lambda_2 = 10 \end{cases}$

Para $\lambda_1 = 12 \rightarrow$ <u>Autovector</u> $= \frac{1}{\sqrt{2}} [1 \;\; 1]^T$

Para $\lambda_2 = 10 \rightarrow$ <u>Autovector</u> $= \frac{1}{\sqrt{2}} [-1 \;\; 1]^T$

<u>Valores singulares</u>: $\begin{cases} \sigma_1 = 2\sqrt{3} \\ \sigma_2 = \sqrt{10} \end{cases}$

$$V = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Por lo tanto $\boxed{\max_{\|x\|=1} \|T(x)\| = 2\sqrt{3}}$ y $\boxed{\min_{\|x\|=1} \|T(x)\| = \sqrt{10}}$

El máximo se alcanza en $x \in S_{\lambda_1=12} = \text{gen}\left\{\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T\right\}$ tal que $\|x\| = 1$.

Entonces, x realiza el máximo si

$$x = \alpha . \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T, \alpha \in \mathbb{R} \text{ y}$$

$$\left\|\alpha \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T\right\| = 1$$

$$\to |\alpha| . \left\|\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T\right\| = 1 \to |\alpha| = 1 \to \alpha = \pm 1$$

Entonces $\boxed{x_{max} = \pm\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T}$

El mínimo se alcanza en $x \in S_{\lambda_2=10} = \text{gen}\left\{\begin{bmatrix}\frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T\right\}$

tal que $\|x\| = 1$

Entonces x realiza el mínimo si

$$x = \alpha . \begin{bmatrix}\frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T, \alpha \in \mathbb{R} \text{ y}$$

$$|\alpha| \left\|\begin{bmatrix}\frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T\right\| = 1 \to |\alpha| = 1 \to \alpha = \pm 1$$

Entonces $\boxed{x_{min} = \pm\begin{bmatrix}\frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T}$